

A Tool for Logicians

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Abstract `turnstile` is a style based on `article.cls` to be used for typesetting articles. Among other uses, the turnstile sign is used by logicians for denoting a consequence relation, related to a given logic, between a collection of formulas and a formula. Many logicians have complaint the lack of a \LaTeX routine for issuing turnstile signs, in any of the forms it could arise.

1 Introduction

Logic is a science whose initial motivation was the analysis of correct reasoning. Nowadays it is very far beyond of studying reasoning, it has many intersections with research areas such as Mathematics, Philosophy, Computer Science, Linguistics, Physics and Artificial Intelligence. One of the main signs used in Logic is turnstile sign, from which there are versions such as “ \vdash ” and “ \models ”, issued respectively by the \LaTeX commands `\vdash` and `\models`

2 `turnstile` Project

The turnstile is a sign used a lot by logicians for denoting a consequence relation, related to a given logic, between a collection of formulas and a formula. Many logicians have complaint the lack of a \LaTeX routine for issuing turnstile signs in any of the forms it could arise, being capable of putting data in the correct places above and below this sign. \LaTeX commands such as `\vdash` and `\models`, although they issue versions of turnstile sign, they are not capable of putting in an adequate way some data below or above them. For example, sometimes it is necessary to place below turnstile sign the name of a considered logical system, and sometimes it is necessary to put above it some additional information.

If we want to say that a formula P is a logical consequence of a collection Γ of formulas in a logic L , we could try to issue it by

$\Gamma \vdash_{\mathrm{L}} P$,

obtaining $\Gamma \vdash_{\mathrm{L}} P$.

Note that “L” was not placed correctly with respect to turnstile sign, but it should be placed and centred exactly below this sign. With `turnstile.sty` we can obtain it by

$\Gamma \sststile{\mathrm{L}}{} P$,

obtaining $\Gamma \sststile{\mathrm{L}}{} P$.

On the other hand, if we want to say that a formula P is a logical semantical consequence of a collection Γ of formulas in a logic L , through varying of x and y ¹, we could try to issue it by

$\Gamma \models_{\mathrm{L}}^{x,y} P$,

obtaining $\Gamma \models_{\mathrm{L}}^{x,y} P$.

Note that both “L” and “ x,y ” were not placed correctly with respect to turnstile sign, they should be placed and centered exactly below and above this sign. With `turnstile.sty` we can obtain it by

$\Gamma \sdtstile{\mathrm{L}}{x,y} P$,

obtaining $\Gamma \sdtstile{\mathrm{L}}{x,y} P$.

The commands provided by this style are all formed by the string “tstile” preceded by a string with two or three letters. These letters indicate the kind of the lines to be successively drawn; they can be “n”, “s”, “d” or “t”. The letter “n” informs that the line is empty, the letter “s” that the line is single, the letter “d” that the line is double, and finally “t” indicates that the line is triple. The corresponding commands for the two lettered strings provide the most common turnstile signs, in which is not drawn a second vertical line following the horizontal line. The first letter of these strings indicates the kind of the vertical line and the second the kind of the horizontal line to be drawn after the vertical line. The three lettered strings can contain any of the letters “n”, “s”, “d” and “t”, with the restriction that the last letter must be distinct from “n”, because the case

1. In [1] and [2] varying objects are presented.

in which the third line is empty is already dealt by the commands with two lettered strings preceding “tstile”. The first letter informs the kind of the first vertical line, the second the kind of the horizontal line, and finally the third letter the kind of the second vertical line.

As you could see above, all these commands have three arguments, whereon the first one is optional.

The first argument is optional, it informs the size by which the internal expressions must be displayed, “d” for displayed formulas, “t” for text formulas, “s” for first subscript or superscript formulas, and “ss” for later subscript or superscript formulas. The default value is “s”. The result of applying “t” or “d” is the same, except if there is a mathematical sign in the second or third argument issued in distinct ways, depending if it is inside text formulas or displayed formulas.

The second and third arguments provide respectively the expressions to be placed below and above the turnstile sign, whereon both these expressions are converted to the size specified by the first argument. On the other hand, if the second or the third argument is empty, then nothing is put below or above the turnstile sign.

3 Examples

Some examples are shown below, whereon, for the sake of illustration, Γ is a given collection of formulas and P is a formula of some logic. Of course, the signs “ Γ ” and “ P ” only illustrate a possible context in which the turnstile sign could appear, among other ones.

`\Gamma \sststyle{}{} P`

$$\Gamma \vdash P \quad (1)$$

`\Gamma \sststyle{\mathrm{LPD}}{} P`

$$\Gamma \vdash_{\mathrm{LPD}} P \quad (2)$$

`\Gamma \sststyle{}{x,y} P`

$$\Gamma \left| \frac{x,y}{} \right. P \quad (3)$$

If the optional argument is not used, then the result is the same as if “s” was the optional argument:

`\Gamma \sststyle{\mathrm{LPD}}{x,y} P`

$$\Gamma \left| \frac{x,y}{\mathrm{LPD}} \right. P \quad (4)$$

`\Gamma \sststyle[d]{\mathrm{LPD}}{x,y} P`

$$\Gamma \left| \frac{x,y}{\mathrm{LPD}} \right. P \quad (5)$$

`\Gamma \sststyle[t]{\mathrm{LPD}}{x,y} P`

$$\Gamma \left| \frac{x,y}{\mathrm{LPD}} \right. P \quad (6)$$

`\Gamma \sststyle[s]{\mathrm{LPD}}{x,y} P`

$$\Gamma \left| \frac{x,y}{\mathrm{LPD}} \right. P \quad (7)$$

`\Gamma \sststyle[ss]{\mathrm{LPD}}{x,y} P`

$$\Gamma \left| \frac{x,y}{\mathrm{LPD}} \right. P \quad (8)$$

$\backslash\Gamma\ \backslash\mathrm{sststyle}\{\mathrm{\mathit{LPDEFGH}}\}\{x,y\}\ P$

$$\Gamma \left| \frac{x,y}{\mathrm{LPDEFGH}} \right. P \quad (9)$$

$\backslash\Gamma\ \backslash\mathrm{sststyle}\{\mathrm{\mathit{LC}}\}\{x,y,z,w\}\ P$

$$\Gamma \left| \frac{x,y,z,w}{\mathrm{LC}} \right. P \quad (10)$$

$\backslash\Gamma\ \backslash\mathrm{sdtstyle}\{\mathrm{\mathit{LC}}\}\{x,y,z,w\}\ P$

$$\Gamma \left| \frac{x,y,z,w}{\mathrm{LC}} \right. P \quad (11)$$

$\backslash\Gamma\ \backslash\mathrm{dststyle}\{\mathrm{\mathit{LC}}\}\{x,y,z,w\}\ P$

$$\Gamma \left\| \frac{x,y,z,w}{\mathrm{LC}} \right. P \quad (12)$$

$\backslash\Gamma\ \backslash\mathrm{ddtstyle}\{\mathrm{\mathit{LC}}\}\{x,y,z,w\}\ P$

$$\Gamma \left\| \frac{x,y,z,w}{\mathrm{LC}} \right. P \quad (13)$$

$\backslash\Gamma\ \backslash\mathrm{dttstyle}\{\mathrm{\mathit{LC}}\}\{x,y,z,w\}\ P$

$$\Gamma \left\| \frac{x,y,z,w}{\mathrm{LC}} \right. P \quad (14)$$

$\backslash\Gamma\mathrm{LC}\{x,y,z,w\}$ P

$$\Gamma \frac{x,y,z,w}{\mathrm{LC}} \big| P \quad (15)$$

$\backslash\Gamma\mathrm{LC}\{x,y,z,w\}$ P

$$\Gamma \frac{x,y,z,w}{\mathrm{LC}} \big| P \quad (16)$$

$\backslash\Gamma\mathrm{LC}\{x,y,z,w\}$ P

$$\frac{x,y,z,w}{\mathrm{LC}} \big\| P \quad (17)$$

$\backslash\Gamma\mathrm{LC}\{x,y,z,w\}$ P

$$\Gamma \frac{x,y,z,w}{\mathrm{LC}} \big\| P \quad (18)$$

$\backslash\Gamma\mathrm{LC}\{x,y,z,w\}$ P

$$\Gamma \frac{x,y,z,w}{\mathrm{LC}} \big\| \big\| P \quad (19)$$

$\backslash\Gamma\mathrm{LC}\{x,y,z,w\}$ P

$$\Gamma \big| \frac{x,y,z,w}{\mathrm{LC}} \big| P \quad (20)$$

`\Gamma \sttstile{\mathrm{LC}}{x,y,z,w} P`

$$\Gamma \left| \frac{x,y,z,w}{LC} \right| P \quad (21)$$

`\Gamma \stttstile{\mathrm{LC}}{x,y,z,w} P`

$$\Gamma \left| \frac{x,y,z,w}{LC} \right| \! \! \! P \quad (22)$$

Below there are given some examples of mathematical expressions below and above turnstile sign which change a lot depending on the optional argument. If no optional argument is given, then it is considered to be “s”.

The reader should also note that the vertical lines don’t stretch according to the height of the expressions located below and above turnstile sign, because we consider that this sign should have a standard height, because logicians use mainly this sign inside texts.

`\Gamma \sststile{\sum_0^{\infty} 1/2^n}{\int_a^b f} P`

$$\Gamma \left| \frac{\int_a^b f}{\sum_0^{\infty} 1/2^n} \right| P \quad (23)$$

`\Gamma \sststile[d]{\sum_0^{\infty} 1/2^n}{\int_a^b f} P`

$$\Gamma \left| \frac{\int_a^b f}{\sum_0^{\infty} 1/2^n} \right| P \quad (24)$$

`\Gamma \sststile[t]{\sum_0^{\infty} 1/2^n}{\int_a^b f} P`

$$\Gamma \left| \frac{\int_a^b f}{\sum_0^{\infty} 1/2^n} \right| P \quad (25)$$

`\Gamma \sststyle[s]{\sum_0^{\infty} 1/2^n}{\int_a^b f} P`

$$\Gamma \left| \frac{\int_a^b f}{\sum_0^{\infty} 1/2^n} P \right. \quad (26)$$

`\Gamma \sststyle[ss]{\sum_0^{\infty} 1/2^n}{\int_a^b f} P`

$$\Gamma \left| \frac{\int_a^b f}{\sum_0^{\infty} 1/2^n} P \right. \quad (27)$$

4 Conclusions

The `turnstile.sty` seems so to be very adequate for issuing the turnstile sign in many ways it can appear, and it puts correctly additional expressions below and above it, if necessary, and stretches the length of the sign as much as it is needed, when some or both expressions are large.

For a future version of this style, we intend to examine the convenience of also changing the height of turnstile sign, so taking into account the heights of the expressions above and below, besides their lengths, as we already do.

References

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